Dependence of the blind deconvolution quality on the input data digitization.

Yulia V.Zhulina

Interstate Joint Stock Corporation “Vympel”, Moscow, Russia.

e-mail: yulia_julina@mtu-net.ru

Abstract.

Analysis of the multi-frame blind deconvolution errors in dependence on the image digitization is given in the article. The general formula for this dependence is received. The detailed results are obtained for images received from a telescope when the input images are formed by the low digitization in optical system of the telescope . As a result at the low bits-depth in the input images the errors of blind deconvolution arise . Comparison between two optical systems having a different quantity of bits per channel is made with the obtained formula . Also the errors in restoration of the real satellites images from telescope are estimated by another more straight method: results of restoration with the 12 bits per channel images are compared with the results received after the input images compression to 8 bits per cannel. The errors estimates received by the real restored images are compared with the results of theoretical formula. This comparison shows a satisfactory agreement of theoretical results with the estimates by real data. Also the analysis of errors shows, that the more frames are processed in the algorithm, the less the input digitization influence on the restoration .

1.Introduction.

In article [1] the algorithm of image restoration by method of blind deconvolution has been offered. This algorithm continues and develops the algorithm published in [2]. The algorithm [2] has been extended in [1] to a case of the multi –frame blind deconvolution. This multi-frame approach gives good results of restoration even in conditions of the strong signal distortions at the telescope aperture[3]. The algorithm was checked at number of different images received by a telescope. It was found out, that a different detailing of image digitization turns out the different quality of image restoration . In this article we estimate errors of restoration in dependence on digitization of the input images from a telescope. Questions of necessary quantity of bits per channel in the image recording arise repeatedly in literature. For example, in [4] the problem is given from the system Photoshop point of view : " Some years ago poor implementation of scanning and digital camera technology caused lots of problems. This problem influenced some experts to recommend working in 16-bits. In the last few years Photoshop image-processing has been refined. Now the high-bit workflows are seldom necessary for editing images in Photoshop." In [5] we can find opinion of the scientific image processing : "Multispectral systems are very computationally intensive because of the greater amount of information to process. Greater bit depth (12-bits or more per channel) is necessary to achieve an appropriate image. Multispectral imaging systems process a greater amount of data than an RGB system. Images produced by multispectral systems are generally not directly viewable, rather, they serve as master images from which derivatives are produced. " The firm "VayTek Inc." specializes in the image deconvolution software and 3D programs. On site [6] this firm gives the following recommendations for choice of cameras: "The noisier the image, the less accurate the deconvolution. It is difficult to compare cameras on this issue. All chips have a bit depth rating from the manufacturer. For example the Sony interline and the Kodak KAF chips are both 12 bit chips. However, the effective bit depth of an image from these two chips is quite different. The Sony chip has a well depth of about 18,000 electrons. Given that it has a readout noise of about 9 electrons per well, the effective signal-to-noise ratio is about 18,000/9 or 2,000. This means the camera can produce an image with about 2000 shades of gray - or an image with a true bit depth of about 11 bits - not the rated 12 bits. On the other hand, the Kodak chip has a well depth of 40,000 electrons and a read noise of about 10 electrons. This gives it a true bit depth of 40,000/10 = 4000 or 12 bits. Visually, the images from these two chips are very similar. However, if you perform an exacting deconvolution process on images from these two cameras, you will see differences in the results" . The necessity of using the higher bit- depths values is explained on site [7] : "Regular 8 and 16 bit per channel applications are simply not capable of realistically or accurately processing real world levels of illumination. HDR floating point images can represent a vastly wide range of values, for example a bright object in a scene might have a brightness of 0.9, and the sun 10,000,000. As a result, you could darken an image, and objects in the scene will become darkened, but the sun is still thousands of times brighter than the rest of the scene".

All the mentioned articles concern basically processing the images received in the Earth conditions or close to a surface of the Earth. However, the problems, connected with observation through the space , are significantly more complex because of the great signal distortions. The first digital recording and numerical reconstruction of optical images in computers date back to 1968-1972 ([8]), and relatively recently computers and optical sensors reached the level high enough to make this application practical ([9-11]).

We show in this article, that solution of the deconvolution task strongly demands for a high quality quantization of images in the output of a telescope. An idea to convert images to 8 bits per channel has a single ground , that a human eye does not require the greater detailing of images. Data from a telescope go through the computational processing. This processing is digital and is not connected with a visual observation . In this article the errors of the multi-frame blind deconvolution are estimated in dependence on digitization of the input images. The general formula for this dependence is received. Also the errors of the real images restoration are calculated and compared with a theoretical formula. At Figs.1-2 the real input images are shown. These are the American satellite Lacrosse-3 (Fig.1) and the Russian satellite Meteor-1(Fig.2).

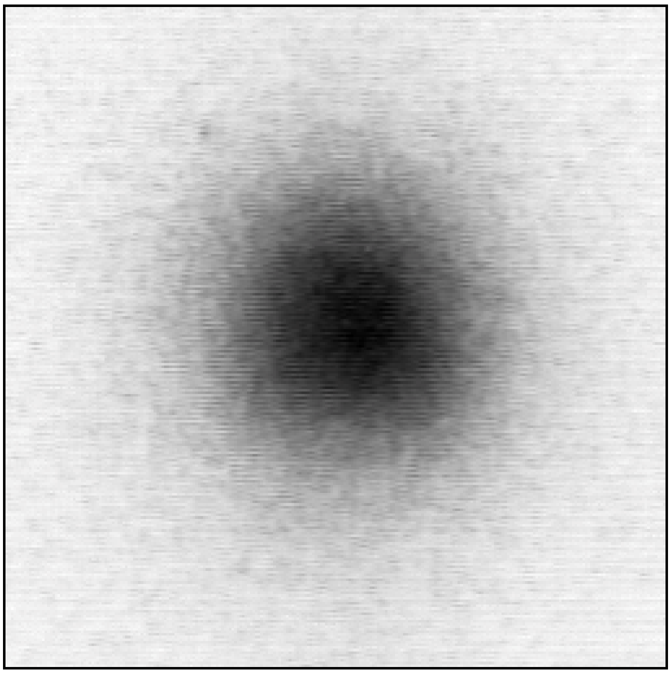

Fig.1 Real image of satellite "Lacrosse-3".

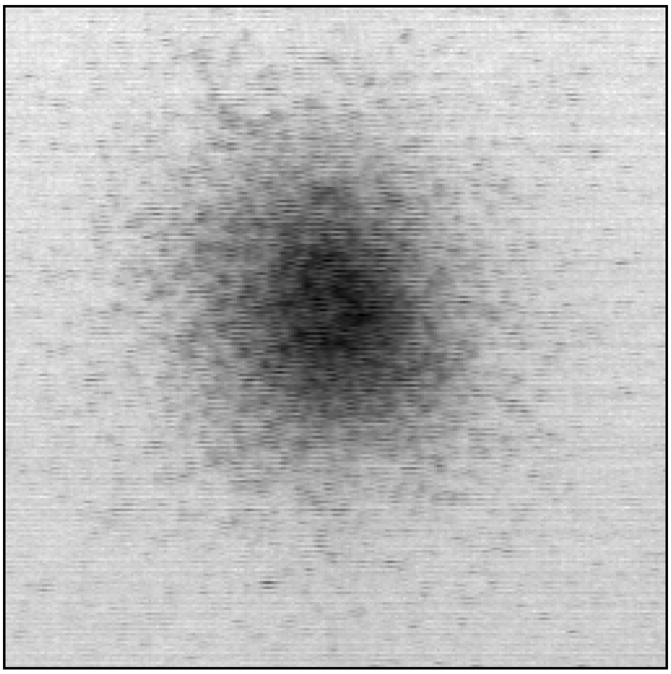

Fig.2 Real image of satellite "Meteor-1".

These images were observed by a telescope in a solar incoherent light. In Figs.3-5 the restored images of Lacrosse-3 and Meteor-1 are shown. They are obtained by the 12 bits/pixel camera. The restored image of Lacrosse-3 with using 19 frames from telescope is shown at Fig.3.

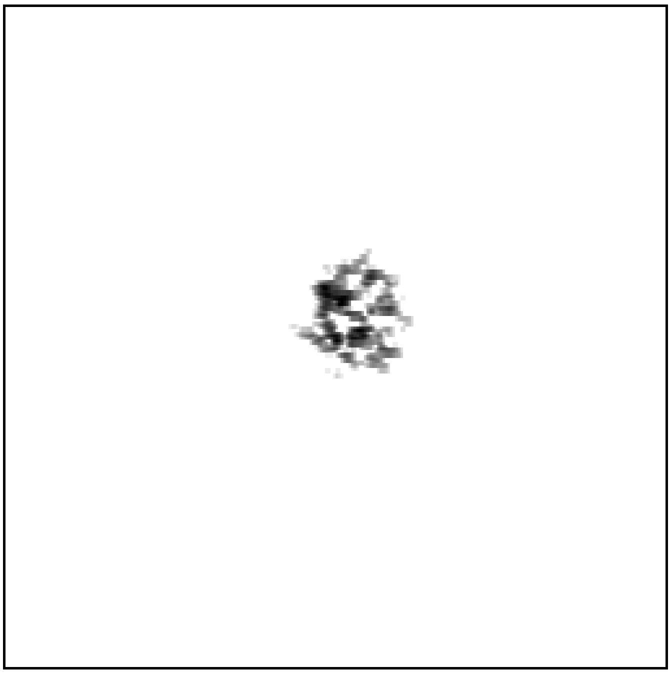

Fig.3 Restoration of satellite “Lacrosse-3”, 61-st iteration,12 bit depth.

At Fig.4 the same restored image can be seen, but some specific elements of the satellite are marked by a name of the element.

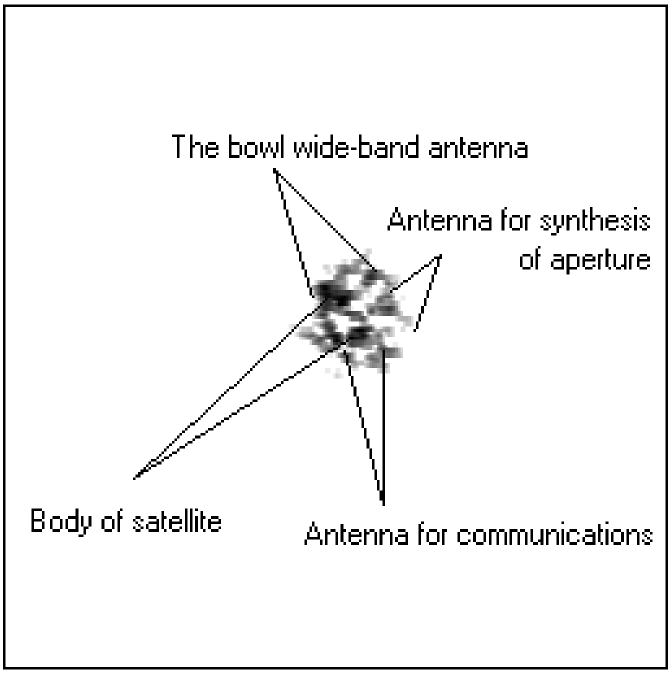

Fig.4 “Restoration of satellite “Lacrosse-3”, 61-st iteration,12 bit depth, comments of details.

At Fig.5 the restored image of the satellite Meteor-1with using 25 frames from a telescope is shown.

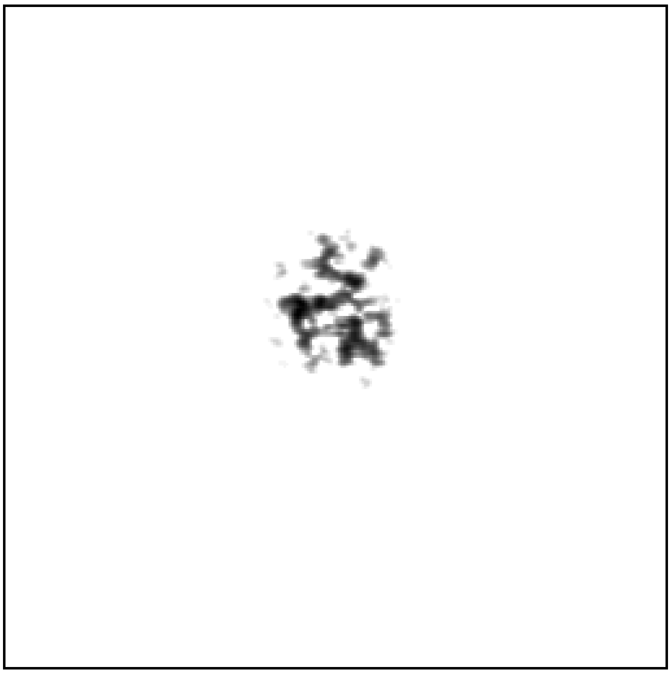

Fig.5 Restoration of satellite “Meteor-1”, 13-th iteration,12 bit depth.

Fig.6 shows a picture of Meteor-1 taken from the Internet.

Fig.6 "Meteor-1" from the Internet.

2. Description of the blind deconvolution algorithm .

Before the task statement we will remind algorithm of deconvolution [1], as influence of image digitization on deconvolution results depends on the algorithm operations. We have M distorted images $O_m(\vec{r})(1 <= m <= M)$ which represent the functions of convolution:

$$O_m(\vec{r}) = \int h_m(\vec{r} - \vec{r}_1)E(\vec{r}_1)d^2\vec{r}_1 + n_m(\vec{r}) \quad (2.1)$$

$O_m(\vec{r})$ - is convolution of $h_m(\vec{r})(1 <= m <= M)$ and $E(\vec{r})$.

Here $h_m(\vec{r})(1 <= m <= M)$ is the unknown and distorted impulse response of optical system , which is new in every frame, $E(\vec{r})$ - the unknown true image which is the same in every frame, $n_m(\vec{r})$- additive noise, also new in every frame. $\vec{r}$ and $\vec{r}_1$ are the vector coordinates of a point in two-dimensional space of the input image. The iterative algorithm of image restoration was offered in [1]. The short description of it is given below.

The following designations are used in the algorithm: $C_m(\vec{\omega})(1 <= m <= M)$- the Fourier transform of the input image $O_m(\vec{r})$, k - number of the iteration cycle , $E^{(k)}(\vec{r})$is an estimate of true image at iteration k before imposing the positivity requirement for estimate, $h_m^{(k)}(\vec{r})$ $m=(1,...,M)$ is an estimate of the optical system impulse response with number m at iteration k before introducing the positivity requirement for it. This requirement should be imposed to all estimates $E^{(k)}(\vec{r})$ and $h_m^{(k)}(\vec{r})$ $m=(1,...,M)$.Therefore we shall enter a designation $E^{(+)(k)}(\vec{r})$ for a positive part of estimate $E^{(k)}(\vec{r})$ and designation $h_m^{(+)(k)}(\vec{r})$ for a positive part of estimate $h_m^{(k)}(\vec{r})$ $m=(1,...,M)$.Also we shall enter designation $G^{(k)}(\vec{\omega})$ for the Fourier transform of $E^{(k)}(\vec{r})$ and $G^{(+)(k)}(\vec{\omega})$ for the Fourier transform of a positive image part $E^{(+)(k)}(\vec{r})$. Here $\vec{\omega}$ is a vector coordinate in the Fourier transform space. Similarly $H_m^{(k)}(\varpi)$ will be a designation for the Fourier transform of $h_m^{(k)}(\vec{r})$ $m=(1,...,M)$ and $H_m^{(+)(k)}(\vec{\omega})$ will design the Fourier transform of a positive part $h_m^{(+)(k)}(\vec{r})$.

The algorithm begins with iteration $k=1$ and initial approximation $E^{(+)(k-1)}(\vec{r})=E^{(+)(0)}(\vec{r})$. The first step is the Fourier transform of $E^{(+)(k-1)}(\vec{r})$ (a step 1). Result of this transform is function $G^{(+)(k-1)}(\vec{\omega})=G^{(+)(0)}(\vec{\omega})$.

The spectrum $H_m^{(k)}(\vec{\omega})$ is calculated by formula (a step 2):

$$H_m^{(k)}(\varpi)=\frac{C_m(\varpi)}{G^{(+)(k-1)}(\varpi)} \qquad (2.2)$$

The inverse Fourier transform of $H_m^{(k)}\ (\vec{\omega})$ gives an estimate $h_m^{(k)}(\vec{r})\ (m=1,...,M)$ (a step 3).The requirement of positivity is imposed on this estimate $h_m^{(k)}(\vec{r})$ and we receive a positive estimate $h_m^{(+)(k)}(\vec{r})\ (m=1,...,M)$ (step 4). Now the Fourier transform of function $h_m^{(+)(k)}(\vec{r})$ is made and the spectrum $H_m^{(+)(k)}(\vec{\omega})\ (m=1,...,M)$ is obtained (a step5). The following step of calculations is the receiving of image spectrum $G^{(k)}(\vec{\omega})$. This spectrum can be obtained by the next formula:

$$G^{(k)}(\vec{\omega}) = \sum_{m=1}^{M} \mu_m^{(k)}(\vec{\omega}) C_m(\vec{\omega}) \qquad (2.3)$$

where $\mu_m^{(k)}(\vec{\omega})$ is :

$$\mu_m^{(k)}(\vec{\omega}) = \frac{H_m^{(+)(k)*}(\vec{\omega})}{\sum_{m=1}^{M} \left| H_m^{(+)(k)}(\vec{\omega}) \right|^2} \qquad (2.4)$$

A proof of formula (2.3) received by method of the maximum likelihood is given in [1].

The operations (2.3 - 2.4) form the step 6. The inverse Fourier transform of (2.3) gives estimate $E^{(k)}(\vec{r})$ (a step 7). Imposing the requirement of positivity to estimate $E^{(k)}(\vec{r})$, we receive a new estimate $E^{(+)(k)}(\vec{r})$ (a step 8). Now $E^{(+)(k)}(\vec{r})$ represents initial approximation for the following iterative cycle. The number of iterations increases for unit , $k = k + 1$, and the all process starts over again.

3.The task statement.

It is necessary to work with numbers of "double" format for image restoration by method of blind deconvolution. These numbers are obtained by transformation from "integer" format to the "double". If the input images $O_m(\vec{r})$ $(m = 1,..., M)$ have a low level of bits/pixel , it means a low quality of the information recording. The image recording with a camera of 8 bits/pixel is the most popular, the cameras of 12 bits/pixel are less often , the 16 bits cameras are seldom and cameras of more than 16 bits/pixel practically are unknown. It means, that images values lie in the range 0-255 in the first case , in the range 0-4095 in the second case, in the range 0-65535 in the third case, and practically we never deal with the full integers.

There is no standard camera that would have enough quantity of bits/pixel and provide a necessary quality of data recording. Therefore it is only possible to compare results with camera, recording K_s bits/pixel (we will consider this quality as an ideal), with results from camera, recording K_d bits/pixel, where $K_d <= K_s$. Any number that is written in space of K_s bits has a range from 0 up to $2^{K_s} - 1$. The quantity of numbers laying in a range of K_s bits exceeds the quantity of numbers that are written in K_d bits to $2^{(K_s - K_d)}$ times. After image compression the number of values i_s laying in the range of K_s bits will be convert to integer number of values i_d ,where:

$$i_d = (\text{int})(i_s \cdot 2^{(K_d - K_s)}) \qquad (3.1)$$

It means, that each number i_s decreases for the value $\Delta(i_s)$, where

$$\Delta(i_s) = i_s - i_d = i_s - (\text{int})(i_s \cdot 2^{(K_d - K_s)}) \qquad (3.2)$$

For example, if $K_d = 8, K_s = 12$,we receive $\Delta(i_s) = i_s - (\text{int})\frac{i_s}{16}$, i.e. 0 remains equal to 0, 15 also turns in 0, 16 turns in 1, 17 turns also in 1, and 256*16 – 1 turns in 255. The value $\Delta(i_s)$

represents a shortage of the pixel value when the "ideal" pixel value is equal to i_s . In the shown example the "ideal" number of bits was $K_s = 12$, but calculation can be made for any K_s. Losses of the information will increase.

Now we will enter designations $dE^{(k)}(\vec{r}), dO_m(\vec{r})$ - accordingly for an errors in the restored image $E^{(k)}(\vec{r})$ (on the k iteration) and for shortages of the input image $O_m(\vec{r})$ with number $m = 1,2...M$. Also we will take the designations for the error correlations function $R_{dE}^{(k)}(\vec{r}_1, \vec{r}_2)$ in restored image and for the shortages correlations function $R_{dO_m}(\vec{r}_1, \vec{r}_2)$ in the input frame with number m :

$$R_{dE}^{(k)}(\vec{r}_1, \vec{r}_2) = < dE^{(k)}(\vec{r}_1) \cdot dE^{(k)}(\vec{r}_2) > \quad (3.3)$$

$$R_{dO_m}(\vec{r}_1, \vec{r}_2) = < dO_m(\vec{r}_1) \cdot dO_m(\vec{r}_2) > \quad (3.4)$$

In the all formulas the sign <A> means averaging of the random variable over the all possible values. Assuming that shortage in the input frame represents a stationary random process, formulas (3.3) and (3.4) can be written so:

$$R_{dE}^{(k)}(\vec{r}_1, \vec{r}_1 - \vec{r}) = R_{dE}^{(k)}(\vec{r}) \quad (3.5)$$

$$R_{dO_m}(\vec{r}_1, \vec{r}_1 - \vec{r}) = R_{dO_m}(\vec{r}) \quad (3.6),$$

where $\vec{r} = \vec{r}_1 - \vec{r}_2$.

In Appendix1 the formula (A1.19) is received, which connects function $R_{dE}^{(k)}(\vec{r})$ with functions $R_{dO_m}(\vec{r}) m = 1,..., M$ .This formula is valid for the various statistics of shortages in the different input frames. If statistical characteristics of shortages in the all frames are identical (this is true for majority of cases), we have:

$$\int_{V_r} R_{dE}^{(k)}(\vec{r})d^2\vec{r} = \frac{2\int_{V_r} R_{dO}(\vec{r})d^2\vec{r}}{\sum_{m=1}^{M}\left(\int_{V_r} h_m^{(+)(k)}(\vec{r})d^2\vec{r}\right)^2} \quad (3.7)$$

Practically this condition is not always kept, as the different frames can statistically differ from each other, for example, shortages can be various because of movement in the orbit. This case is not considered in this article . If statistical characteristics of shortages are identical in the different frames(formula (A1.21)), the correlation functions are identical in the all frames . In this case index m can be omitted in

$R_{dO_m}(\vec{r})$ (formula 3.7). In (3.7) and everywhere below V_r designates the space of coordinates $\vec{r}$ in the image, and V_ϖ designates the space of its spectrum values $\vec{\varpi}$.

Now it is necessary to calculate integral in the right part of (3.7) . It is difficult to do it precisely , but it is possible to estimate a level of errors in a vicinity of true images. We will suppose, that the all impulse responses are calculated approximately correctly at the end of iterative process, i.e. the estimates $h_m^{(+)(k)}(\vec{r})$ $(m = 1,...,M)$ approximately coincide with true impulse response $h_m(\vec{r})$. In this case it is possible to neglect dependence $R_{dE}^{(k)}(\vec{r})$ on number k at the end of iterative process and we can write formula (3.7) in a view:

$$\int_{V_r} R_{dE}(\vec{r})d^2\vec{r} = \frac{2\int_{V_r} R_{dO}(\vec{r})d^2\vec{r}}{\sum_{m=1}^{M} I_m^2} \qquad (3.8),$$

where

$$I_m = \int_{V_r} h_m(\vec{r})d^2\vec{r} = \frac{1}{A}\int_A U_m^2(\vec{\omega})d^2\vec{\omega} \qquad (3.9)$$

Here $U_m(\vec{\omega})$ is function of the amplitude distortions in the spectrum coordinates $\vec{\omega}$, A is the space of coordinates $\vec{\omega}$ at the optical aperture .

The value I_m is calculated in Appendix2 (formula (A2.5)), and represents a square of the signal amplitude distortions averaged over the area of the aperture . The atmosphere never strengthens a signal which passes through it but only weakens its amplitude and leads to attenuation of the signal. Therefore it is possible to suppose, that value $I_m <= 1$, i.e. is practically always less than unit.

Obviously , (3.8) describes the image errors correlations only when the iterative process is already stabilized, and the estimates of image and impulse responses weakly change with increasing of iteration number. It is follows from (3.8) , that the stronger amplitude distortions are,(i.e. the less values of coefficients I_m are) , the larger integral of the image errors is. Also the more number of frames M is used, the less influence of the input frames shortages on the restoration .

4. Approximation of errors in the restored image.

Now we have to receive the correlation function $R_{dO}(\vec{r})$ for (3.8) . We will suppose that any values $0 <= i_s < 2^{K_s}$ can appear in any pixel of the input image. In this assumption it is possible to receive a shortage variance $(\sigma_\Delta)^2$ as an average square deviation of $\Delta(i_s)$ (3.2) by formula:

$$\sigma_\Delta^2 = \sum_{i_s=0}^{i\max-1} \Delta^2(i_s) \cdot P(i_s) \qquad (4.1)$$

where $i\max = 2^{K_s}$, (4.2)

$P(i_s)$ is a probability of the pixel value equal to i_s. We will enter a designation:

$$I_0 = 2^{(K_s - K_d)} \qquad (4.3)$$

We can write the next approximation of $\Delta(i_s)$ (3.2) :

$$\Delta(i_s) = i_s \cdot \alpha \quad \text{if} \quad i_s >= I_0 \qquad (4.4)$$

where

$$\alpha = \frac{2^{K_s} - 2^{K_d}}{2^{K_s}} \quad , \qquad (4.5),$$

and

$$\Delta(i_s) = i_s \quad \text{if } i_s < I_0 \qquad (4.6).$$

(4.6) means, that if we use the image with K_d bit/pixel instead of image with K_s bits/pixel the all pixels with values $i_s < I_0$ become equal to zero. It does not mean removal of additive noise in the image: after the pixel nullifying the details with a small intensity will be lost without possibility of restoration.

Now we will suppose that the all possible values of background intensity are distributed with equal probability in the interval from 0 up to S_F. Also we will assume, that the all possible values of the object intensity are distributed with equal probability in interval from 0 up to S_I .Using (4.4-4.6), it is possible to rewrite (4.1):

$$\sigma_\Delta^2 = \sum_{i_s=0}^{I_0-1} i_s^2 \cdot P(i_s) + \alpha^2 \cdot \sum_{i=I_0}^{i\max-1} i_s^2 \cdot P(i_s) \qquad (4.7)$$

The maximal possible value of pixel satisfies to condition:

$$i\max = 2^{K_s} - 1 = S_I + S_F \qquad (4.8),$$

and value I_0 is accordingly equal:

$$I_0 = 2^{(K_s - K_d)} \approx \frac{S_I + S_F}{2^{K_d}} \qquad (4.9)$$

Calculation (4.7) gives the different results for the different maximum levels of background: for the big levels of background ($S_F >= S_I$) and for the small levels of a background: $S_F << S_I$. If the telescope observes in the normal weather conditions , the second variant is more real. We will consider only the second case. Probabilities $P(i_s)$ of the pixel value i_s are calculated in Appendix3. Calculations by formula (4.7) lead to a rather large formula. Taking in account values of the largest order in (4.7) we can receive formula for σ_Δ :

$$\sigma_\Delta \approx \alpha \cdot \frac{2^{K_s}}{\sqrt{6}} = \frac{1}{\sqrt{6}}(2^{Ks} - 2^{Kd}) \qquad (4.10)$$

The restriction $S_F << S_I$ means, that the maximal value of the object image much more exceeds the maximal value of background in the frame. From expression (4.10) it is visible, that if $K_s = K_d$ variance (4.10) is equal to zero, and in the all other cases ,when $K_d < K_s$, the variance will be rather big. If $K_s = 12; K_d = 8;$ we have $\sigma_\Delta \approx 1672$. Value σ_Δ is really visible only in the compressed frame with $K_d = 8$ and , accordingly to formula (3.1) in compressed image σ_Δ will be equal to:

$$\sigma_{dO} = (\text{int})(\sigma_\Delta \cdot 2^{(K_d - K_s)}) \approx 104 \qquad (4.11)$$

From (4.11) it is visible, that transformation of the two-bytes array to the one-byte array leads to the noise equal to 41 % from the maximal value in the frame with 8 bits/pixel. In this case the level of background in the initial input frame of 12bits/pixel before compression can be very low . All this leads to deterioration of the restored images. On a simple example it is possible to be convinced, how the compression to one-byte file from the two-byte file worsens the resolution of optical system. Assume that the two-bytes frame contains the peak with a value equal to 511 , and it is surrounded by 8 points of values equal to 256. After compression to the one-byte file the all nine points will become the identical values equal to 1. In the two-byte file the center stands out obviously , so the resolution is equal to one pixel . After compression to the one-byte file and receiving a spot constituted of 9 identical points equal to 1 ,the resolution is worsened three times on both coordinates.

Using formula (4.11), it is possible to calculate an average deviation of the image restoration because of compression to one-byte data array. For simplifying we will use formula (3.8) in assumption of absence of the amplitude distortions. Assuming, that all errors in the right and in the left parts of (3.8) are uncorrelated , we receive formula for the errors variance in pixel of the restored image:

$$\sigma_{dE} = \sigma_{dO} \sqrt{\frac{2}{M}} \qquad (4.12)$$

If $\sigma_{dO} = 104; M = 19$, we receive: $\sigma_{dE} \approx 33.9$.It amounts to approximately 13 percents of the maximum level in the one-byte image. If $\sigma_{dO} = 104; M = 25$, we receive : $\sigma_{dE} \approx 29.5$. It amounts to approximately 11 percents of the maximum level in the one-byte image.

5. Calculation errors in the real restored images.

For comparison we can estimate the real errors after replacement of the real images with 12 bits/pixel with images with 8 bits/pixel. For this purpose there was made restoration (Fig.3) of the satellite Lacrosse-3 by 19 frames that were received with camera of 12 bits/pixel (Fig.1) . Then these initial images were compressed to the 8bits/pixel , and restoration also was made (Fig.7).

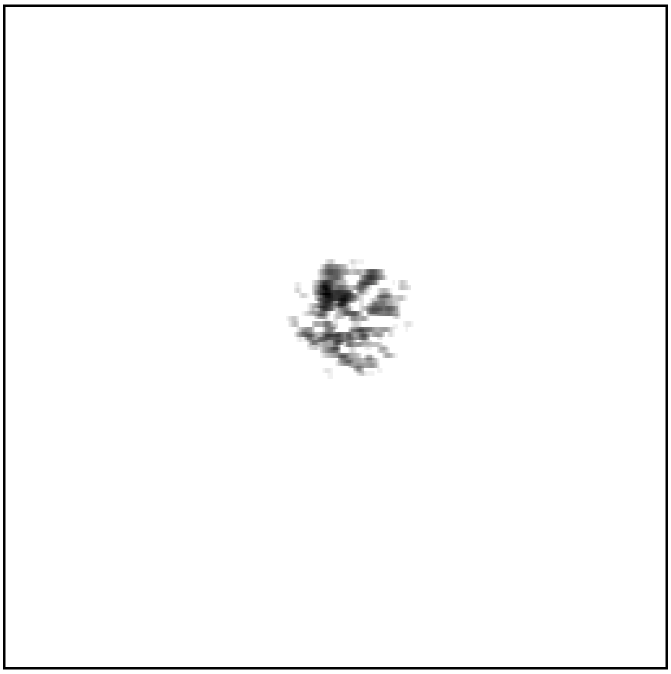

Fig.7 Restoration of satellite "Lacrosse-3", 60-th iteration,8 bit depth..

Deterioration of restoration by the one-byte images in comparison with restoration by the 12 bits images was estimated in such way: the image array Ar_1 restored by the 12 bits images was considered as an ideal, and the image array Ar_2 restored by the one-byte images was considered as the result with errors. $Ar_1[i], Ar_2[i]$ $(i = 0,...N_x \cdot N_y)$ -are the values of the restored pixels with number i in the space of frame, and N_x, N_y - are the sizes of frame along x, y - accordingly. The square deviation of one array from another was calculated and then it was normalized to quantity of pixels in the frame :

$$\sigma_{real}^2 = \sum_{i=0}^{N_x \cdot N_y - 1} \frac{(Ar_2[i] - Ar_1[i])^2}{N_x \cdot N_y} \qquad (5.1)$$

The value σ_{real} is considered as error of restoration by the one-byte arrays in comparison with the 12-bits arrays, analogous to the error (4.12) σ_{dE} . The calculations were made by formula (5.1) with using images at Fig.3 and Fig.7. Dimensions $N_x = N_y = 256$, number of processed frames $M = 19$, number of iteration $k = 61$, $\sigma_{real} = 31.3$. This result coincides good enough with the error $\sigma_{dE} \approx 33.9$ calculated by formula (4.12). Also there was made restoration (Fig.5) of the satellite Meteor-1-1 with 25 frames received by 12 bits camera(Fig.2). After this the input frames have been compressed to the one-byte frames, and restoration was made with them (Fig.8).

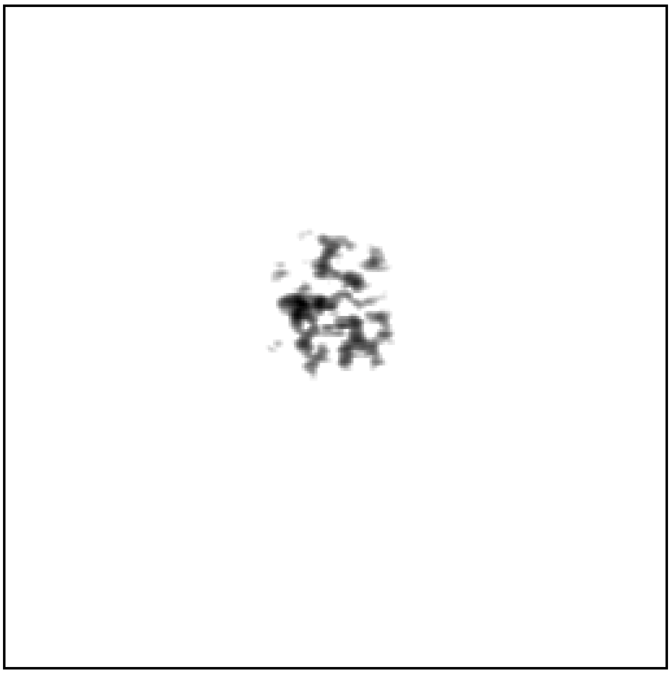

Fig.8 Restoration of satellite “Meteor-1”, 14-th iteration,8 bit depth..

The calculations were made by formula (5.1) with using images at Fig.5 and Fig.8. Dimensions $N_x = N_y = 256$, number of frames $M = 25$, number of iteration $k = 13$, the error $\sigma_{real} = 22.6$. In this case the coincidence with error $\sigma_{dE} \approx 29.5$ (4.12) turned out some worse .

Conclusion.

We investigated dependence of the blind deconvolution results on digitization of the convolution recording . The general formulas (A1.19) - (A1.21) for the restoration errors are obtained . These formulas connect the correlation function of deconvolution errors with the correlation function of the loosed bits in the input frames. The errors of restoration are estimated by formula (A1.21) in a supposition that the record of images with K_s bits/pixel gives the ideal quality of digitization, and the record with K_d bits/pixel

($K_d <= K_s$) leads to errors in the image restoration. Calculations were made for a case $K_s = 12$ and $K_d = 8$.

Also the errors were calculated for the real data from a telescope . Only frames of 12bits- depth have been at the disposal for calculations. Restoration with images of 12 bits/pixel was considered as the test specimen and the work with 8 bits/pixel was considered as restoration with errors . This calculation was made on the frames received from satellites " Lacrosse-3 " and "Meteor-1". Calculation shows a satisfactory agreement of theoretical results (formula (A1.21)) with the estimates by real data. This agreement is better, the better is restoration of the test specimen. For "Lacrosse-3" 12 bits /pixel appeared to be enough, and agreement of theoretical results with the real ones is good. For "Meteor-1" the agreement appeared to be worse. It is necessary to note, that the more quantity of frames is processed , the less the final errors of restoration are. It speaks once more for the multi-frame blind deconvolution.

So the deep digitization of input data for blind deconvolution is necessary. The useful information in the single byte images can be completely nullified or absorbed by a noise in the pixels having a low level of intensity. On recording more than one byte this information is not nullified ,it can be successfully processed by the multi-frame deconvolution algorithm. If we process the sufficient number of frames we can restore more details which have a low intensity .

Appendix1. Calculation the errors correlation function in the restored image.

Formula (2.3) calculates the image spectrum $G^{(k)}(\varpi)$ at iteration with number k.

Differential $dG^{(k)}(\varpi)$ of function (2.3) has a view:

$$dG^{(k)}(\varpi) = \sum_{m=1}^{M}(d\mu_m^{(k)}(\varpi)\cdot C_m(\varpi) + \mu_m^{(k)}(\varpi)\cdot dC_m(\varpi)) \quad \text{(A1.1)}$$

$dG^{(k)}(\varpi), d\mu_m^{(k)}(\varpi), dC_m(\varpi)$ are the designations for differentials of the image spectrum $G^{(k)}(\vec{\omega})$, coefficient $\mu_m^{(k)}(\vec{\omega})$ (2.4) and the input image spectrum $C_m(\vec{\omega})$ with number m accordingly. These differentials can be considered as errors in the vicinity of true values $G^{(k)}(\vec{\omega})$, $\mu_m^{(k)}(\vec{\omega})$, $C_m(\vec{\omega})$. The spectral density [12-13] $S_{dG}^{(k)}(\vec{\omega})$ of restored spectrum errors can be written trough these differentials :

$$S_{dG}^{(k)}(\vec{\omega}) =< \left|dG^{(k)}(\vec{\omega})\right|^2 >=< \left|\sum_{m=1}^{M}(d\mu_m^{(k)}(\vec{\omega})\cdot C_m(\vec{\omega}) + \mu_m^{(k)}(\vec{\omega})\cdot dC_m(\vec{\omega}))\right|^2 > \qquad \text{(A1.2)}$$

The all errors in the frames with different numbers m can be considered as the uncorrelated random values . Then it is possible to rewrite (A1.2) in a following view:

$$S_{dG}^{(k)}(\vec{\omega}) = \sum_{m=1}^{M} (S_{d\mu_m}(\vec{\omega})\cdot\left|C_m(\vec{\omega})\right|^2 + S_{dC_m}(\vec{\omega})\cdot\left|\mu_m^{(k)}(\vec{\omega})\right|^2 + 2\,\mathrm{Re}(S_{dC_m d\mu_m{}^{(k)*}}(\vec{\omega})\cdot C_m^*(\vec{\omega})\cdot\mu_m^{(k)}(\vec{\omega}))) \qquad \text{(A1.3)}$$

where

$$S_{d\mu_m}(\vec{\omega}) =< \left|d\mu_m^{(k)}(\vec{\omega})\right|^2 > \qquad \text{(A1.4)}$$

is the spectral density [12-13] of the coefficients (2.4) errors,

$$S_{dC_m}(\vec{\omega}) =< \left|dC_m(\vec{\omega})\right|^2 > \qquad \text{(A1.5)}$$

is the spectral density [12-13] of the input spectrum errors,

$$S_{dC_m d\mu_m{}^{(k)*}}(\vec{\omega}) =< dC_m(\vec{\omega})d\mu_m^{*(k)}(\vec{\omega}) > \qquad \text{(A1.6)}$$

is the spectral density of two errors correlation : the input spectrum error and the error of coefficient (2.4) , both of number m .

Taking in account (2.2) and (2.4) and keeping in (A1.3) only values of the largest order, we can receive instead of (A1.3) the next expression:

$$S_{dG}^{(k)}(\varpi) = \frac{2 \cdot \sum_{m=1}^{M} S_{dC_m}(\varpi) \cdot \left| H_m^{(+)(k)}(\varpi) \right|^2}{\left(\sum_{m=1}^{M} \left| H_m^{(+)(k)}(\varpi) \right|^2 \right)^2} \qquad \text{(A1.7)}$$

For further calculations we will enter designations:

$$F_{A_m^{(k)}}(\varpi) = \left| H_m^{(+)(k)}(\varpi) \right|^2 \qquad \text{(A1.8),}$$

$$F_{B^{(k)}}(\varpi) = \sum_{m=1}^{M} \left| H_m^{(+)(k)}(\varpi) \right|^2 \qquad \text{(A1.9),}$$

$$F_{D^{(k)}}(\varpi) = (F_{B^{(k)}}(\varpi))^2 \qquad \text{(A1.10)}$$

and designation for the inverse Fourier transform:

$$A_m^{(k)}(\vec{r}) = \frac{1}{2\pi} \int_{V_\omega} F_{A_m^{(k)}}(\varpi) \exp(j\varpi \cdot \vec{r}) d^2\varpi \qquad \text{(A1.11)}$$

Using the inverse Fourier transform for (A1.8) - (A1.10), we can receive:

$$A_m^{(k)}(\vec{r}) = \int_{V_r} h_m^{(+)(k)}(\vec{r}_1) h_m^{(+)(k)}(\vec{r}_1 - \vec{r}) d^2\vec{r}_1 \qquad \text{(A1.12)}$$

$$B^{(k)}(\vec{r}) = \sum_{m=1}^{M} A_m^{(k)}(\vec{r}) \qquad \text{(A1.13)}$$

$$D^{(k)}(\vec{r}_1) = \int_{V_r} B^{(k)}(\vec{r}) B^{(k)}(\vec{r}_1 - \vec{r}) d^2\vec{r} \qquad \text{(A1.14)}$$

In formulas (A1.11) - (A1.14) V_r is the space of coordinates $\vec{r}$, and V_{ϖ} is the space of ϖ.After substitution (A1.8 - A1.10) to(A1.7) we can receive:

$$S_{dG}^{(k)}(\varpi)\cdot F_{D^{(k)}}(\varpi)=2\cdot\sum_{m=1}^{M}S_{dC_m}(\varpi)\cdot F_{A_m^{(k)}}(\varpi) \quad \text{(A1.15)}$$

It is known [12-13] , that the errors correlation function $R_{dE}^{(k)}(\vec{r})$ (3.5) can be received by the inverse Fourier transform from the spectral density $S_{dG}^{(k)}(\varpi)$:

$$R_{dE}^{(k)}(\vec{r})=\frac{1}{2\pi}\int_{V_\omega}S_{dG}^{(k)}(\varpi)\exp(j\varpi\cdot\vec{r})d^2\varpi \quad \text{(A1.16)}$$

Function of shortages correlations $R_{dO_m}(\vec{r})$ (3.6) in the input frame can be received similarly:

$$R_{dO_m}(\vec{r})=\frac{1}{2\pi}\int_{V_\omega}S_{dC_m}(\vec{\omega})\exp(j\vec{\omega}\cdot\vec{r})d^2\vec{\omega} \quad \text{(A1.17)}$$

Now we will take the inverse Fourier transform from the both parts of (A1.15). Using the designations (3.5)-(3.6) , we can receive:

$$\int_{V_r}R_{dE}^{(k)}(\vec{r})D^{(k)}(\vec{r}_1-\vec{r})d^2\vec{r}=2\cdot\sum_{m=1}^{M}\int_{V_r}R_{dO_m}(\vec{r})A_m^{(k)}(\vec{r}_1-\vec{r})d^2\vec{r} \quad \text{(A1.18)}$$

Expression (A1.18) is valid at any point of space $\vec{r}_1$. Integrating (A1.18) over $\vec{r}_1$ and taking in account (A1.14), we receive instead of (A1.18):

$$\int_{V_r} R_{dE}^{(k)}(\vec{r})d^2\vec{r} = \frac{2\sum_{m=1}^{M}(I_m^{(k)})^2 \int_{V_r} R_{dO_m}(\vec{r})d^2\vec{r}}{(\sum_{m=1}^{M}(I_m^{(k)})^2)^2} \qquad \text{(A1.19)}$$

In (A1.19) a designation for integral from the impulse response estimate is entered:

$$I_m^{(k)} = \int_{V_r} h_m^{(+)(k)}(\vec{r})d^2\vec{r} \qquad \text{(A1.20)}$$

If statistical characteristics are identical in the all input frames, (A1.19) becomes more simple . In this case the function $R_{dO_m}(\vec{r})$ is identical in the all frames and index m can be omitted. Then we receive:

$$\int_{V_r} R_{dE}^{(k)}(\vec{r})d^2\vec{r} = \frac{2\int_{V_r} R_{dO}(\vec{r})d^2\vec{r}}{\sum_{m=1}^{M}(\int_{V_r} h_m^{(+)(k)}(\vec{r})d^2\vec{r})^2} \qquad \text{(A1.21)}$$

Appendix2. The impulse response of the incoherent optical system at presence of the amplitude and phase distortions.

The impulse response of the optical incoherent system $h(\vec{r})$ has a view [12-13] :

$$h(\vec{r}) = \left|V(\vec{r})\right|^2 \qquad \text{(A2.1)}$$

where function $V(\vec{r})$ is:

$$V(\vec{r}) = \frac{1}{\sqrt{A}\cdot l\lambda}\int_A \exp(jk/l\cdot\vec{\omega}\cdot\vec{r} + j\varphi(\vec{\omega}))d^2\vec{\omega} \qquad \text{(A2.2)}$$

In (A2.2) $k = \dfrac{2\pi}{\lambda}$ is the wave number, A is the space of coordinates $\vec{\varpi}$ at the optical aperture , $\varphi(\vec{\varpi})$ is function of the phase distortions in coordinates $\vec{\varpi}$.

Formula (A2.2) is valid if the amplitude distortions are absent. In presence of the amplitude distortions we have instead of formula (A2.2):

$$V(\vec{r}) = \frac{1}{\sqrt{A} \cdot l\lambda} \int_A \exp(jk / l \cdot \varpi \cdot \vec{r} + j\varphi(\varpi)) U(\varpi) d^2\varpi \quad \text{(A2.3)}$$

Here $U(\vec{\varpi})$ is function of the amplitude distortions in coordinates $\vec{\varpi}$.

Through expression (A2.3) the impulse response is expressed also by formula (A2.1). In amplitude distortions we receive from A2.1:

$$\int_{V_{\vec{r}}} h(\vec{r}) d^2\vec{r} = \frac{1}{A} \int_A U^2(\varpi_1) d^2\varpi_1 \qquad \text{(A2.4)}$$

Otherwise, integral of the impulse response over the frame is equal to the square of the amplitude distortions averaged over the aperture.

If the amplitude distortions are absent, $U(\vec{\varpi}) = 1$ and formula (A2.4) gives :

$$\int_{V_{\vec{r}}} h(\vec{r}) d^2\vec{r} = 1 \quad \text{(A2.5)}$$

Appendix3. Probability distribution of the pixel values in the input image .

The probability distribution $P(i)$ of the pixel values i in the input image can be written , using that the designations $i, P(i), S_F, S_I$:

$$P(i) = \frac{1}{S_F S_I} \int_0^{S_F} \int_0^{S_I} \delta(i - x - y) dx dy \qquad \text{(A3.1)}$$

Here x, y are designations of the noise and the object image values accordingly .

We suppose that the all possible values of noise intensity are distributed with equal probability in the interval from 0 up to S_F . Also we will assume, that the all possible values of the object image intensity are distributed with equal probability in interval from 0 up to S_I . δ is an one-dimensional delta-function. Taking integral over y , we will receive instead of (A3.1):

$$P(i) = \frac{1}{S_F S_I} \int_{\max(0, i - S_I)}^{\min(S_F, i)} dx \qquad \text{(A3.2)}$$

Keeping the values of the largest order in (A3.2), it is possible to receive:

$$P(i) = \frac{i}{S_F S_I}, \text{ if } 0 <= i <= S_F \qquad \text{(A3.3)}$$

$$P(i) = \frac{1}{S_I}, \quad \text{if } S_F <= i <= S_I \qquad \text{(A3.4)}$$

$$P(i) = \frac{S_F + S_I - i}{S_F S_I}, \text{ if } S_I <= i <= S_I + S_F \quad \text{(A3.5)}$$

References:

[1] . Y.V.Zhulina,"Multiframe blind deconvolution of heavily blurred astronomical images ",Applied Optics, v.45, №28, October, 2006.

[2] G. R. Ayers and J. C. Dainty, “Iterative blind deconvolution method and its applications,” Opt. Lett., vol. 13, no. 7, pp. 547–549, July 1988.

[3] http://www.imagerestoration.lgg.ru

[4] Jim Rich,“The PhotoShop High-Bit Advantage: Fact or Fiction”, Apr2003, Digital Output .

[5] Donald S. Brown , Eastman Kodak Company ,"Image Capture Beyond 24-Bit RGB", RLG DigiNews, Vol 3, No. 3, 1999, http://www.imaging.org/resources

[6] http://www.vaytek.com

[7] http://www.vaytek.com/comparecameras.asp

[8] J.M. Goodman and R.W.Lawrence, "Digital image formation from electronically detected holograms", Applied Physics Letters, vol.11,no.3, pp.77-79, 1967.

[9]Y. Wang, J. Ostermann, and Y.Q.Zhang, "Video Processing and Communications," 1st ed., Prentice Hall; ISBN: 0130175471, 2002.

[10] L. Yaroslavsky, "Digital Holography and Digital Image Processing", Kluwer Academic Publishers, Boston, 2003.

[11] Rafael C. Gonzalez, Richard E.Woods, "Digital Image Processing", 3rd Edition, Prentice Hall; ISBN: 013168728X; August 2007.

[12] J. Goodman, "Introduction to Fourier Optics", McGraw-Hill, New York, NY, USA,1996.

[13] J.W. Goodman, "Statistical Optics", John Wiley & Sons, (1985).